NASA-CR-194764

NAGW-2918
IN-93-CR
198578
9P

Relativistic Shock Spectra: A Prediction

J. I. Katz

Department of Physics and McDonnell Center for the Space Sciences

Washington University, St. Louis, Mo. 63130

I: katz@wuphys.wustl.edu

Abstract

I argue that particles heated by relativistic shocks should assume an equilibrium energy distribution. This leads to a synchrotron spectrum $F_\nu \propto \nu^{1/3}$ up to approximately the critical frequency ν_0 of an electron with the mean electron energy. Application to GRBs implies that a burst with 10^{-5} erg/cm^{2}s of soft gamma-rays and $h\nu_0 = 300$ KeV should be about 18th magnitude in visible light and a few μJy at 1 GHz (less if self-absorbed).

(NASA-CR-194764) RELATIVISTIC SHOCK SPECTRA: A PREDICTION (Washington Univ.) 9 p

N94-21782

Unclas

G3/93 0198578

Subject headings: Gamma-rays: Bursts—Shock Waves—Acceleration of Particles

1. Introduction

A model of gamma-ray bursts (GRB) has been proposed in which a relativistic fireball's debris shell (Shemi and Piran 1990) forms a relativistic collisionless shock when it interacts with surrounding low-density gas (Rees and Mészáros 1992, Mészáros and Rees 1993a, Katz 1994a). The shock-heated plasma radiates by the synchrotron process, producing the observed gamma-rays. It has been usual in astrophysics to assume that shock-accelerated particles have a power-law distribution in energy. In this paper I argue that this assumption is inapplicable to relativistic shocks, for which I predict a thermal equilibrium particle distribution function and a synchrotron spectrum $F_\nu \propto \nu^{1/3}$ below its peak.

2. Fermi Acceleration

Most particle acceleration mechanisms considered in astrophysics (with the exception of acceleration in a large quasi-static electric field in pulsars) are based on developments of Fermi's (1949) model of stochastic acceleration. These mechanisms include acceleration by a nonrelativistic shock (Blandford and Eichler 1987), turbulent plasma acceleration (Sagdeev and Galeev 1969) and viscous shear acceleration (Katz 1991). In these processes the accelerated particles are not in thermal equilibrium, and the evolution of their distribution function is described by a Fokker-Planck equation.

Fermi derived the general result that the distribution function produced by these stochastic acceleration processes is proportional to a power of the particle energy, but a dimensional argument is sufficient: Any deviation from a power law would define a characteristic energy, yet in these models of stochastic acceleration there are no characteristic energies between a particle's rest mass and an extremely high energy above which it is not magnetically confined to the region of acceleration. The number density n of accelerated particles is indeterminate because there is an essentially infinite reservoir of (nonrelativistic) thermal particles. The energy density e of accelerated particles is also indeterminate

because the acceleration is driven by a reservoir of fluid kinetic energy in the shock or plasma or hydrodynamic turbulence, an indeterminate fraction of which is converted to particle acceleration. The prediction of a power-law differential distribution of particle energy under these conditions

$$N(E) \propto E^{-p}, \tag{1}$$

where $N(E)$ is the number of particles per unit energy per unit volume, is borne out by observations of cosmic rays and of power-law synchrotron spectra radiated by accelerated particles

$$F_\nu \propto \nu^{-s} \tag{2}$$

in many astronomical objects; in optically thin synchrotron theory (Rybicki and Lightman 1979) $s = (p-1)/2$ if $p > 1/3$.

In order that the total number and energy of accelerated particles be finite, a distribution of the form (1) must have at least one characteristic energy E_0 at which p changes, with $p = p_< < 1$ for $E < E_0$ and $p = p_> > 2$ for $E > E_0$. The corresponding synchrotron spectral indices $s_< < 0$ and $s_> > 1/2$. In most astronomical synchrotron sources (but not GRB) E_0 is very small (the rest mass energy, or the injection energy below which Coulomb drag is significant) and outside the range of observation, and the spectral index $s_< < 0$ is not observable (synchrotron radiation by such low energy particles is negligible or at unobservably low frequencies, and the particles may be nonrelativistic, for which different results apply). Observed spectral indices usually exceed 1/2 (for cosmic rays the observed $p > 2$), as expected for $E > E_0$.

3. Relativistic Shocks

It is often assumed that a power-law particle distribution function (Eq. 1) will apply to any collisionless gas of shock-accelerated particles. This is incorrect for relativistic

collisionless shocks*. In such a shock n and e are determined by the jump conditions, and the mean energy per particle sets the natural energy scale $E_0 = e/n$.

In order for a shock to occur there must be an irreversible process, which mixes the distribution function in phase space, increasing a coarse-grained entropy. Without detailed knowledge of the mechanics of this process, it is natural to assume that it leads to a final state in which all microstates consistent with the constraints (on n and e) are equally probable. This assumption defines a microcanonical ensemble (Reif 1965), and was also made by Lynden-Bell (1967) in his discussion of violent gravitational relaxation. The resulting distribution of particle energies, assuming relativistic kinematics and nondegeneracy, is that of thermal equilibrium

$$N(E) \propto E^2 \exp(-3E/E_0); \tag{3}$$

the "temperature" $k_B T = E_0/3$, and the parameters $p_< = -2$ and $p_> \rightarrow \infty$. This argument for (3) depends essentially on the existence of the constraints.

In a collisionless shock (3) must result from the interaction of coarse-grained clumps in phase space, rather than from single-particle collisions. These clumps interact electromagnetically through plasma turbulence, and we may regard the fields as the means by which the clumps interact, in analogy with the gravitational interaction of coarse-grained clumps considered by Lynden-Bell.

If the phase space clumps are sufficiently long-lived (as, for example, may be the clumps discussed by Dupree [1982]), they may come to equilibrium with each other and with the turbulent fields, considered as independent degrees of freedom. This hypothesis is distinct from that of violent relaxation. The brightness temperature of the turbulence

* The argument presented by Katz (1994a) for $p = 3/2$ and $s = 1/4$ is wrong; nonrelativistic shock acceleration theory is inapplicable, and the assumed compression ratio is inconsistent with the (correct) values cited elsewhere in that paper.

is given by

$$k_B T_b = \frac{\mathcal{F}_\nu c^2}{\nu^2}, \tag{4}$$

where $\mathcal{F}_\nu$ is the spectral density of the turbulence. In a relativistic plasma most modes will have relativistic phase and group velocities, so we may approximate

$$\mathcal{F}_\nu \approx \frac{B^2 c}{8\pi \Delta\nu}, \tag{5}$$

where B is the turbulent magnetic field and $\Delta\nu$ is its spectral bandwidth. A clump of size λ will contain $\approx n\lambda^3$ particles and a total energy $\approx n\lambda^3 E_0 \approx k_B T_c$, where T_c is the kinetic temperature of the clumps. Taking $\Delta\nu \approx \nu$ and $\lambda \approx c/\nu$ and equating T_c and T_b yields

$$e = nE_0 \approx \frac{B^2}{8\pi}. \tag{6}$$

This demonstrates that equipartition between clumps and plasma turbulence is consistent with the energetics, and lends credibility to the equipartition of particle and magnetic energy assumed by Katz (1994a).

The length λ is determined by the wavelengths of the fastest growing plasma instabilities. These may be two-stream instabilities when the plasmas first interpenetrate, and then velocity space anisotropy instabilities. Equation (6) also implies approximate equality between the plasma and gyro-frequencies ω_p and ω_g, so that the distribution of λ is probably peaked around $\lambda \approx 2\pi c/\omega_p \approx 2\pi c/\omega_g$, with $\mathcal{F}_\nu$ peaked around $\nu \approx c/\lambda$.

Electron-ion equipartition cannot result from the very slow single-particle interactions, and must result from electromagnetic coupling between charge- or current-unneutralized clumps in electron and ion phase space. These complex processes are beyond the scope of this paper.

Finally, a nonequilibrium distribution can also be shown to relax to the equilibrium (3) if an H-theorem is satisfied, which follows from the assumption of molecular chaos (Liboff 1969). This assumption holds if the phase space clumps interact weakly, but is likely to be more general—H-theorems are observed to hold in dense and strongly coupled systems for

which it is hard to justify the assumption of molecular chaos. Strong two-particle (or two-clump) momentum correlations are disrupted, at least in ensemble average, as the particles (or clumps) interact with many others, and thermodynamic equilibrium is observed even though the assumption of molecular chaos is hard to defend.

4. Predicted Spectra

For electron distributions with $p < 1/3$ the synchrotron spectrum at frequencies below the critical frequency is dominated by the radiation of the most energetic electrons (Jackson 1975), and $s = -1/3$ (rather than $(p-1)/2$). Thus the particle distribution (3) leads to a predicted radiation spectrum

$$F_\nu \propto \nu^{1/3}. \tag{7}$$

This power law spectrum survives averaging over emission regions with a range of E_0, magnetic field, and Doppler shift if the frequency of observation is everywhere below the critical synchrotron frequency (Doppler-shifted to the observer's frame) for electrons with the local E_0 in the local field. The observation (Schaefer 1994) in GRB at soft X-ray energies of $N_\nu \propto \nu^{-0.7}$, corresponding to $F_\nu \propto \nu^{0.3}$, may be confirmation of (7). The steeper spectra of GRB at harder X-ray and gamma-ray energies may reflect evolving heterogeneous source regions with a distribution of (Doppler-shifted) critical synchrotron frequencies, some of which are below the frequency of observation.

In GRB the spectrum (7) should extend from X-rays to radio frequencies, where self-absorption becomes significant (Mészáros and Rees 1993b, Paczyński and Rhoads 1993, Katz 1994ab). Because the radiation at all frequencies is dominated by electrons with $E \approx E_0$, in the self-absorbed region $s = -2$ rather than the value $s = -5/2$ found (Rybicki and Lightman 1979) when $p > 1/3$.

The spectrum of GRB may be normalized to the observed soft gamma-ray fluxes. The normalization is necessarily very approximate, because source regions are likely heterogeneous and the transition between (7) and the expected exponential cutoff at high photon

energies may therefore extend over several decades of the integrated spectrum, including the soft gamma-ray region. An intense GRB with a flux 10^{-5} erg/cm^{2}s in a bandwidth of 400 KeV around $h\nu = 300$ KeV has a flux there of ≈ 10 mJy. Extrapolation leads to a flux of ≈ 0.2 mJy in visible light, (an $\approx$ 18th magnitude star) and to ≈ 2 μJy at 1 GHz. The effective flux of a brief transient measured by a broad-band receiver may be further reduced by plasma dispersion. Because these fluxes are low (and, in addition, self-absorbed at low frequencies) intergalactic dispersion (Ginzburg 1973, Palmer 1993, Katz 1994ab) and optical counterparts to GRB will be difficult to observe. For an actual GRB it is more accurate to extrapolate downward from the highest frequency ν_0 at which (7) is observed, which may lead to substantially higher visible and radio fluxes if $s > -1/3$ between ν_0 and soft gamma-ray frequencies.

After the initial gamma-ray transient, a relativistic fireball will continue to expand and to produce radiation up to a critical frequency which decreases with time as the blast wave degrades (Paczyński and Rhoads 1993, Katz 1994ab, Mészáros and Rees 1993b, 1994). In a uniform medium (a very unrealistic assumption, as shown by the complex temporal structure of observed GRB) the intensity at a given frequency will increase with time (in a simple model $\propto t^{4/5}$; Katz 1994b) until the critical synchrotron frequency decreases to the frequency of observation. However, at all times the spectrum (7) should be observed in the very broad range of frequencies between the critical frequency and the onset of self-absorption.

5. Discussion

This paper makes two predictions for the characteristic spectrum of synchrotron radiation produced by relativistic shocks. The first prediction, $s_< < 0$, uses only the finiteness of the total particle number. The second, more specific, prediction, $s_< = -1/3$ (7), is based upon the argument (§3) for an equilibrium particle distribution function (3), though any distribution with $p < 1/3$ is sufficient to lead to $s = -1/3$.

The equilibrium particle distribution (3) also predicts an exponential cutoff on the radiation spectrum at high frequency. This has not generally been observed in GRB, which may be explained if the observed radiation is the superposition of emission from a distribution of emitting regions with a wider range of parameters and characteristic synchrotron energies, extending up to the the highest photon energies observed. In addition, because of the poor counting statistics at high photon energies, particularly when there is a rapid cutoff, it is hard to verify the functional form of the cutoff.

The arguments presented here for relativistic shocks are general, and not specific to GRB. For example, they should apply to relativistic blast wave models of AGN such as those proposed by Blandford and McKee (1977).

I thank P. Diamond and B. E. Schaefer for discussions and NASA NAGW-2918 for support.

References

Blandford, R. D. & Eichler, D. 1987 Phys. Rep. 154, 1

Blandford, R. D. & McKee, C. F. 1977 MNRAS 180, 343

Dupree, T. H. 1982 P. Fluids 25, 277

Fermi, E. 1949 PRev 75, 1169

Ginzburg, V. L. 1973 Nature 246, 415

Jackson, J. D. 1975 Classical Electrodynamics (Wiley, New York)

Katz, J. I. 1991 ApJ 367, 407

Katz, J. I. 1994a ApJ 422, in press

Katz, J. I. 1994b Huntsville Gamma-Ray Burst Workshop, eds. G. Fishman, K. Hurley, J. Brainerd (AIP, New York) in press

Liboff, R. L. 1969 Introduction to the Theory of Kinetic Equations (Wiley, New York)

Lynden-Bell, D. 1967 MNRAS 136, 101

Mészáros, P. & Rees, M. J. 1993a ApJ 405, 278

Mészáros, P. & Rees, M. J. 1993b ApJ 418, L59

Mészáros, P. & Rees, M. J. 1994 Huntsville Gamma-Ray Burst Workshop, eds. G. Fishman, K. Hurley, J. Brainerd (AIP, New York) in press

Paczyński, B. & Rhoads, J. E. 1993 ApJ 418, L5

Palmer, D. M. 1993 ApJ 417 L25

Rees, M. J. & Mészáros, P. 1992 MNRAS 258, 41p

Reif, F. 1965 Fundamentals of Statistical and Thermal Physics (McGraw-Hill, New York)

Rybicki, G. B. & Lightman, A. P. 1979 Radiative Processes in Astrophysics (Wiley, New York)

Sagdeev, R. Z. & Galeev, A. A. 1969 Nonlinear Plasma Theory (Benjamin, New York)

Schaefer, B. E. 1994 Huntsville Gamma-Ray Burst Workshop, eds. G. Fishman, K. Hurley, J. Brainerd (AIP, New York) in press

Shemi, A. & Piran, T. 1990 ApJ 365, L55